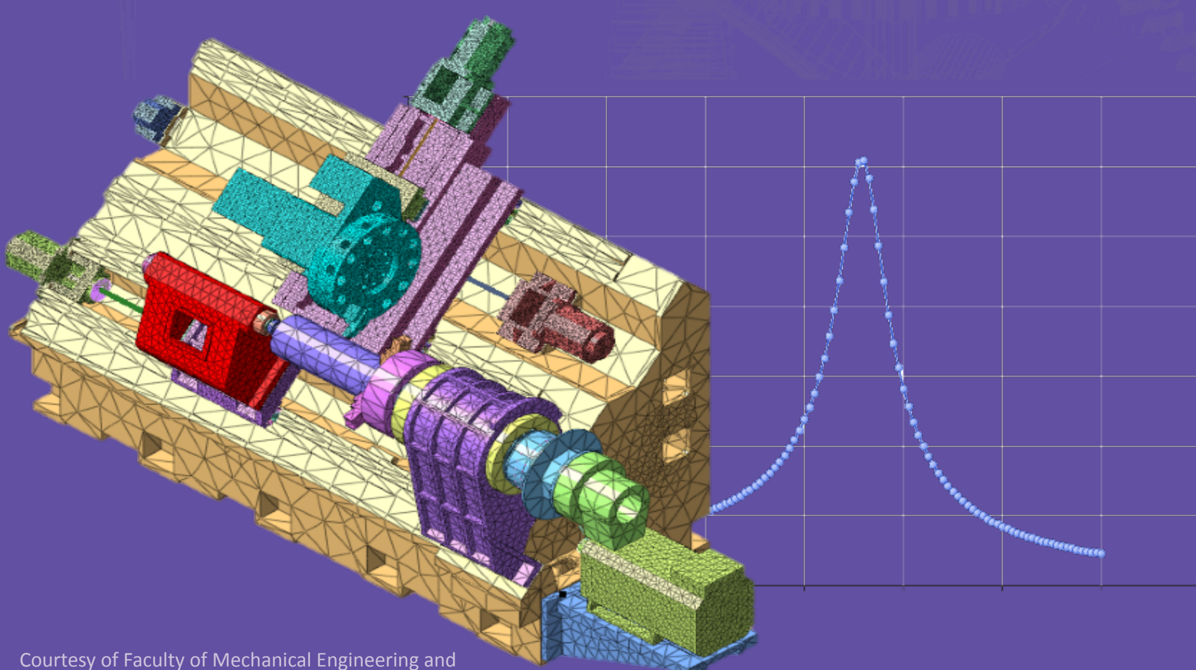
FEA forAll
From basics to Advanced

A Guide to linear dynamic analysis with Damping

This guide starts from the applications of linear dynamic response and its role in FEA simulation. Fundamental concepts and principles will be introduced such as equations of motion, types of vibration, role of damping in engineering, linear dynamic analyses, etc.

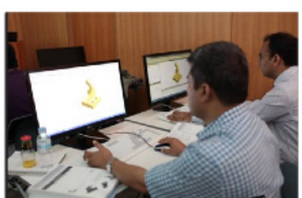


Courtesy of Faculty of Mechanical Engineering and Mechatronics; West Pomeranian University of Technology, Poland

About the Author:

My name is Cyprien Rusu, I am a French CAE engineer who wants to teach the right bases of FEA Simulation to designers, engineers and everyone aspiring to get it right!
Hundreds of FEA students followed my free FEA webinars on Youtube, read my blog articles on feaforall.com and joined my FEA courses to learn more and improve their understanding of FEA and become better engineers!

I have also taught FEA seminars to FEA engineers from all over the world…



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1. Dynamic Analysis Application

Dynamic analysis is strongly related to vibrations.

Vibrations are generally defined as fluctuations of a mechanical or structural system about an equilibrium position. Vibrations are initiated when an inertia element is displaced from its equilibrium position due to an energy imported to the system through an external source.

Vibrations occur in many mechanical and structural systems. Without being controlled, vibrations can lead to catastrophic situations.

Vibrations of machine tools or machine tool chatter can lead to improper machining of parts. Structural failure can occur because of large dynamic stresses developed during earthquakes or even wind induced vibrations.

Figure 1: 1940 Tacoma Narrows Bridge failure

Vibrations induced by an unbalanced helicopter blade while rotating at high speeds can lead to the blade's failure and catastrophe for the helicopter. Excessive vibrations of pumps, compressors, turbo-machinery, and other industrial machines can induce vibrations of the surrounding structure, leading to inefficient operation of the machines while the noise produced can cause human discomfort.

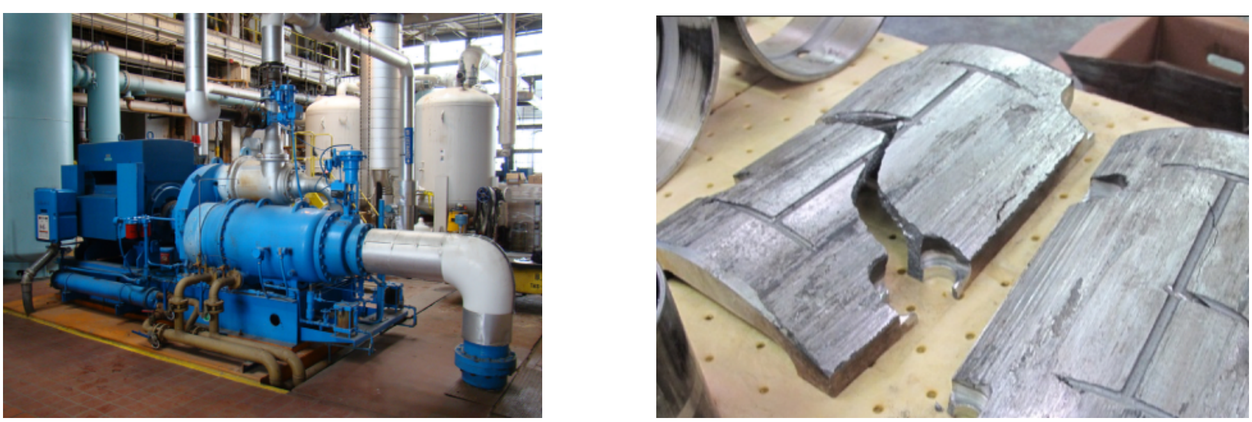


Figure 2: Failed compressor components

Vibrations as the science is one of the first courses where most engineers to apply the knowledge obtained from mathematics and basic engineering science courses to solve practical problems. Solution of practical problems in vibrations requires modeling of physical systems. A system is abstracted from its surroundings. Usually assumptions appropriate to the system are made.

Basic engineering science, mathematics and numerical methods are applied to derive a computer based model.

2. Dynamic Analysis Equation

The mathematical modeling of a physical system results in the formulation of a mathematical problem. The modelling is not complete until the appropriate mathematics is applied and a solution obtained.

The type of mathematics required is different for different types of problems. Modeling of any statics, dynamics, and mechanics of solids problems leads only to algebraic equations. Mathematical modeling of vibrations problems leads to differential equations.

In mathematical physics, equations of motion are equations that describe the behavior of a physical system in terms of its motion as a function of time.

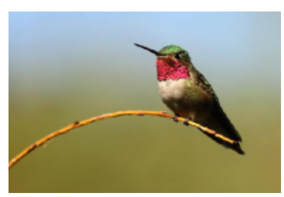
Equations of motion are consisted of **inertial force, damping force (energy dissipation) and elastic (restoring) force.**

The **overall behavior of a structure** can be grasped through these three forces.

$$m\ddot{u}(t) \quad + \quad c\dot{u}(t) \quad + \quad ku(t) \quad = \quad p(t)$$

INERTIA FORCE **DAMPING FORCE** **RESTORING FORCE** **APPLIED FORCE**






Inertia Force is generated by accelerated mass.

Damping Force describes energy dissipation mechanism which induces a force that is a function of a dissipation constant and the velocity. This force is known as the general viscous damping force.

The final **induced force** in the dynamic system is due to the elastic resistance in the system and is a function of the displacement and stiffness of the system. This force is called the elastic force, restoring force or occasionally the spring force.

The **applied load** has been presented on the right-hand side of equation and is defined as a function of time. This load is independent of the structure to which it is applied.

Exact analytical solutions, when they exist, are preferable to numerical or approximate solutions. Exact solutions are available for many linear problems, but for only a few nonlinear problems.

2.1 Single Degree of Freedom System

Simple mechanical system is schematically shown in Figure 3. The inputs (or excitation) applied to the system are represented by the force p(t). The outputs (or response) of the system are represented by the displacement u(t). The system boundary (real or imaginary) demarcates the region of interest in the analysis. What is outside the system boundary is the environment in which the system operates.

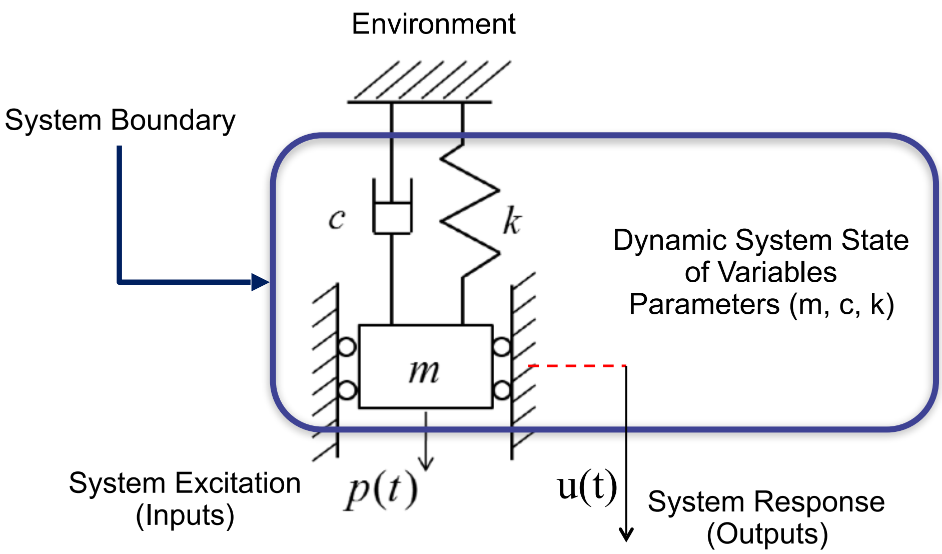


Figure 3: A Mechanical dynamic system

System parameters are represented in the model, and their values should be known in order to determine the response of the system to a particular excitation.

State variables are a minimum set of variables, which completely represent the dynamic state of a system at any given time t. For a simple SDOF oscillator an appropriate set of state variables would be the displacement u and the velocity du/dt.

The equation of motion for SDOF mechanical system may be derived using the free-body diagram approach.

2.2 Equation of Motion for Single Degree of Freedom System

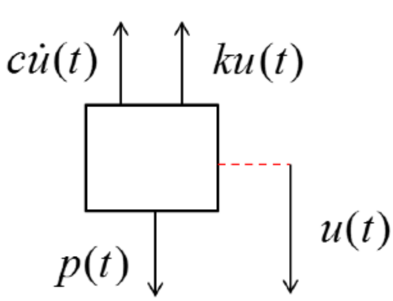


Figure 4: SDOF free body diagram.

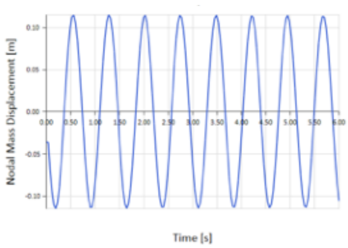
$$\boxed{p(t) = m\ddot{u}(t) + c\dot{u}(t) + ku(t)}$$

m – mass
c – damping coefficient
k – stiffness coefficient
p(t) – applied force

u(t)– mass displacement
$\dot{u}$(t)– mass velocity
$\ddot{u}$(t)– mass acceleration

2.3 Single Degree of Freedom System Responses

Free Undamped Response



$$m\ddot{u}(t) + ku(t) = 0$$

General solution:

$$u(t) = A\cos\omega_n t + B\sin\omega_n t$$

$$u(t) = \frac{\dot{u}_0}{\omega_n}\sin\omega_n t + u_0\cos\omega_n t$$

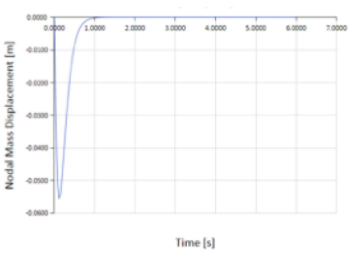
$\dot{u}_0$ and u_0 are an Initial Condition entities

Natural frequency: $\omega_n = \sqrt{\frac{k}{m}}\ [\frac{rad}{s}]$

Free Damped Response

$$m\ddot{u}(t) + c\dot{u}(t) + ku(t) = 0$$

$c = c_{cr} = 2\sqrt{km} = 2m\omega_n$

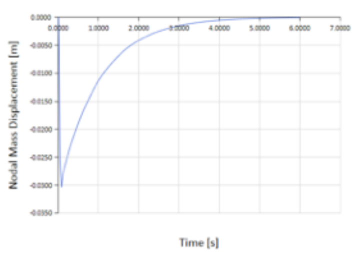


Critically Damped System

General solution:

$$u(t) = (A + Bt)e^{-ct/2m}$$

$c > c_{cr}$

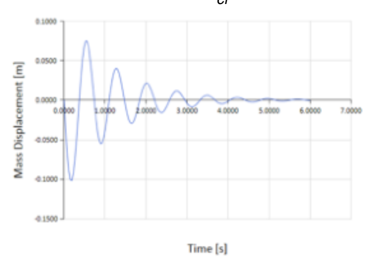


Over-damped System

Underdamped solution:

$$u(t) = e^{-ct/2m}(A\sin\omega_d t + B\cos\omega_d t)$$

$c < c_{cr}$

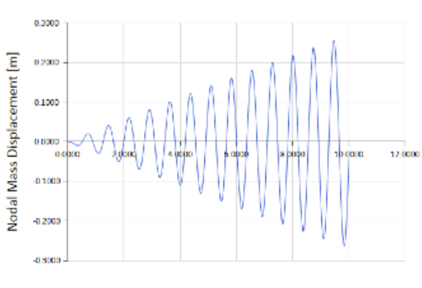


Underdamped System

$\omega_d = \omega_n\sqrt{1-\zeta^2}$ Damped natural frequency

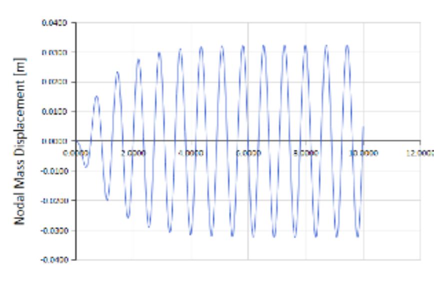
$\zeta = \frac{c}{c_{cr}}$ Damping ratio

Forced Undamped Response



$$m\ddot{u}(t) + ku(t) = p(t)$$

Forced Damped Response

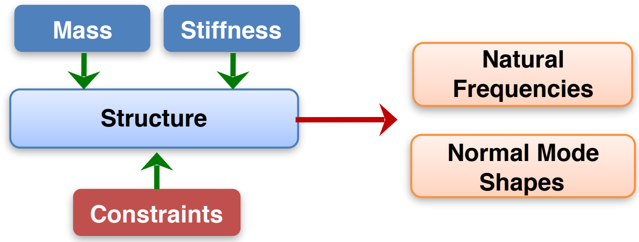


$$m\ddot{u}(t) + c\dot{u}(t) + ku(t) = p(t)$$

3. Dynamic Analysis Types

3.1 Eigenvalue Analysis/Normal Modes/Modal

Analysis of the **Normal Modes** or **Natural Frequencies** of a structure is a **search** for it's **resonant frequencies.** By understanding the dynamic characteristics of a structure experiencing oscillation or periodic loads, we can prevents resonance and damage of the structure.

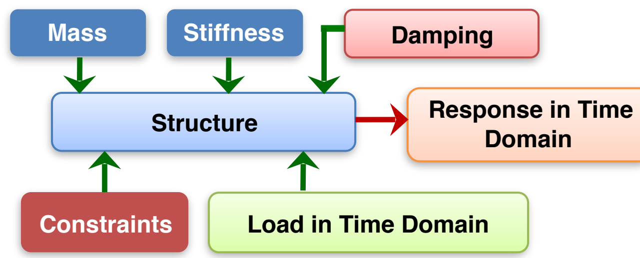


Natural Frequency – the actual measure of frequency, [Hz] or similar units
Normal Mode shape – the characteristic deflected shape of a structure as it resonates

3.2 Transient Analysis

Executed in the time domain, it obtains the solution of a dynamic equation of equilibrium when a dynamic load is being applied to a structure.

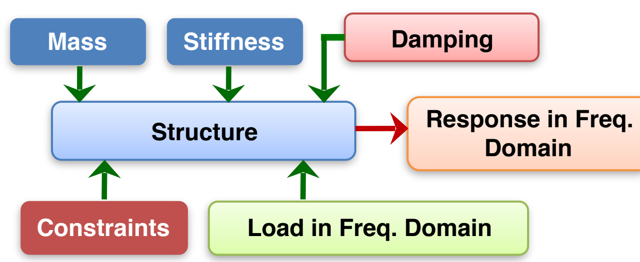
Though the load and boundary conditions required for a transient response analysis are similar to those of a static analysis, a difference is that load is defined as a **function of time**.



3.3 Frequency Response Analysis

A technique to determine the **steady state response** of a structure according to sinusoidal (har*monic*) loads of known frequency.

Frequency Response is best **visualised as a response** to a structure **on a shaker table**. Adjusting the frequency input to the table gives a **range of responses.**



$$m\ddot{u}(t) + c\dot{u}(t) + ku(t) = F\sin(\omega t)$$

$$u(t) = F/k \frac{\sin(\omega t + \theta)}{\sqrt{\left(1 - \omega^2/\omega_n^2\right)^2 + \left(2\zeta\omega/\omega_n\right)^2}}$$

ω – input or driving frequency φ – phase lead

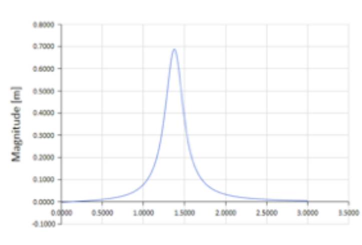


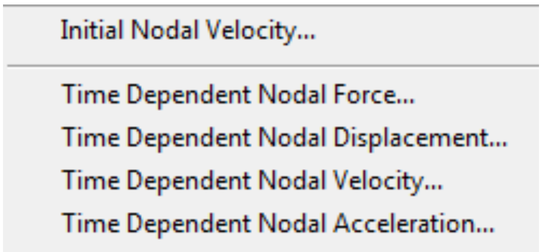
Figure 5: SDOF frequency response - Displacement

Figure 6: Shaking table for frequency response experiment

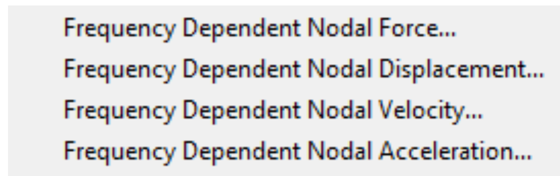
4. Dynamic Loading Types

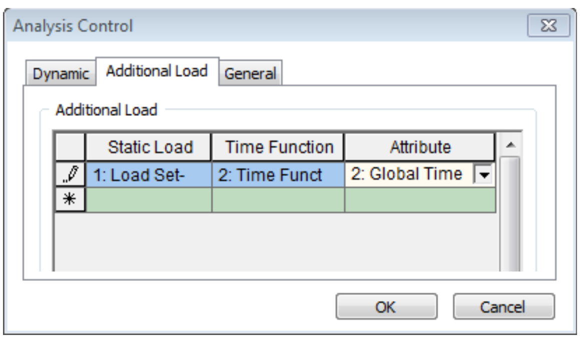
Dynamic loads can be applied directly on your model or as time or frequency dependent static loadings.

- Time domain based loading types



- Frequency domain based loading types





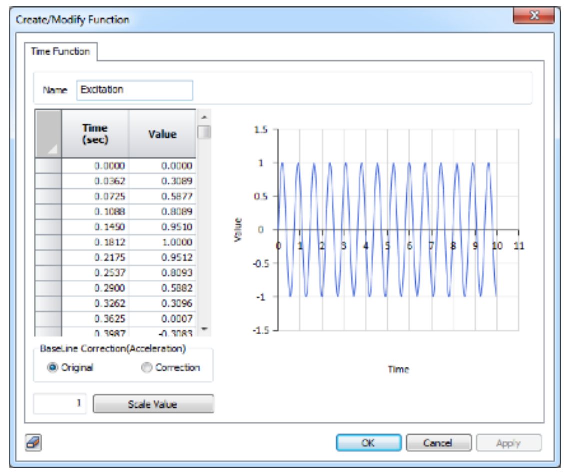
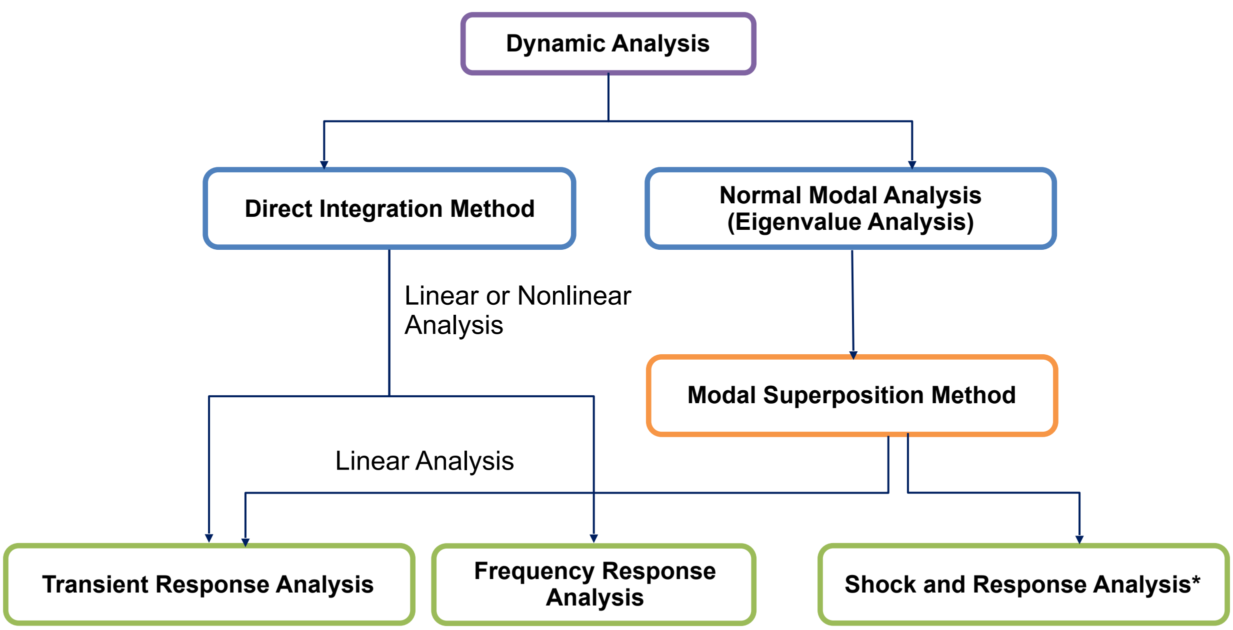


Figure 7. Domain dependent Static Loading.

4.1 Solution Methods



*Not Covered in this guide

Figure 8. Dynamic Analysis Solution Types and Methods.

5. Damping

Damping is the phenomenon by which mechanical energy is dissipated (usually by conversion into internal thermal energy) in dynamic systems. Knowledge of the level of damping in a dynamic system is important in the utilisation, analysis, and testing of the system. In structural systems, damping is more complex, appearing in several forms.

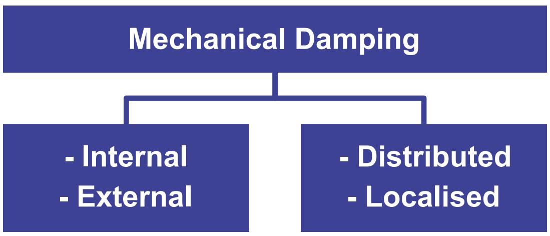


Figure 9. Damping forms

Internal damping refers to the structural material itself. Internal (material) damping results from mechanical energy dissipation within the material due to various microscopic and macroscopic processes.

External damping comes from boundary effects. An important form is structural damping, which is produced by rubbing friction: stick-and-slip contact or impact. Another form of external damping is fluid damping.

Figure 10. Localised damping examples

All damping ultimately comes from frictional effects, which may however take place at different scales. If the effects are distributed over volumes or surfaces at macro scales, we speak of distributed damping. Damping devices designed to produce beneficial damping effects, such as shock absorbers, represent localised damping.

5.1 Damping Models

It is not practical to incorporate detailed microscopic representations of damping in the dynamic analysis of systems. Instead, simplified models of damping that are representative of various types of energy dissipation are typically used.

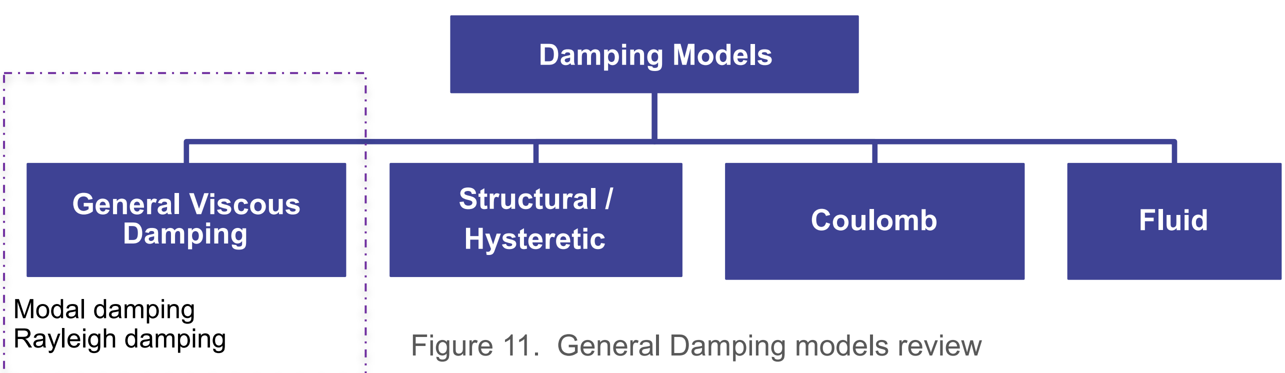


Figure 11. General Damping models review

5.2 Damping Models

There are many types of damping:

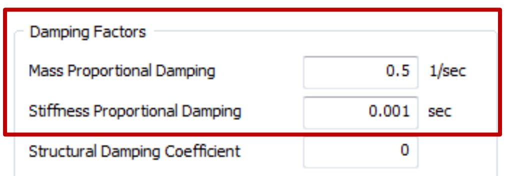
Proportional Damping (Rayleigh; classic)

- Hysteretic/Structural Damping
- Direct Damping values
- Frequency dependent damping
- Modal Damping
- Coulomb damping, requires special modelling techniques

5.3 Damping Model input for:

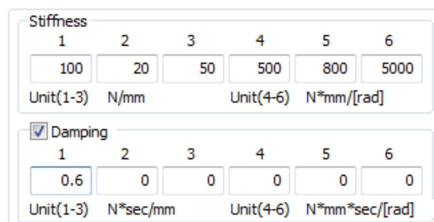
- Rayleigh Damping (Proportional)

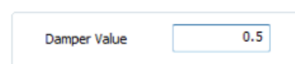
- Defined via Material Card



- Discrete Viscous Damping

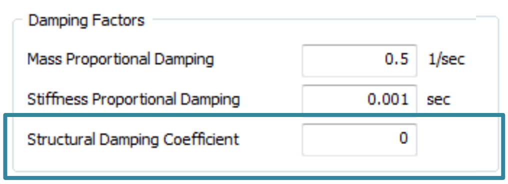
- BUSH 1D element Property
- Damper element Property

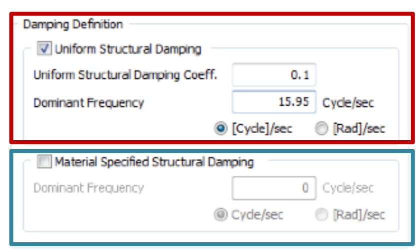




- Structural Damping

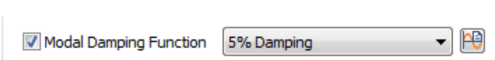
- Defined via Material Card and Analysis Case Manager
- Overall and Elemental Damping

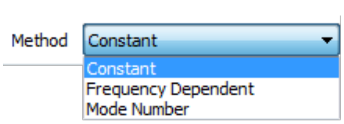


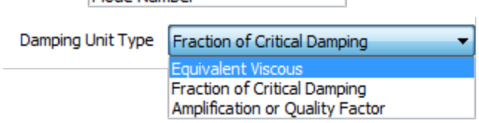


- Modal Damping

- Defined via Analysis Case Manager and Functions

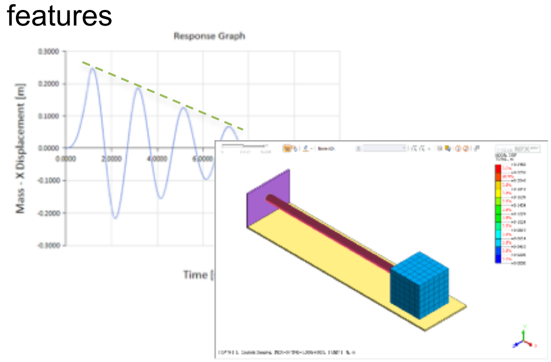






- Coulomb Damping

- Defined by combination of elements and features



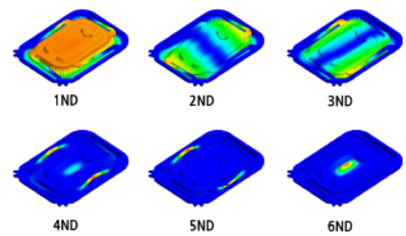
6. Dynamic Analysis Project Applications

Various projects require dynamic analysis. Here are a few examples:

Case 1

Performance evaluation of a mobile speaker through sound pressure level (SPL) analysis

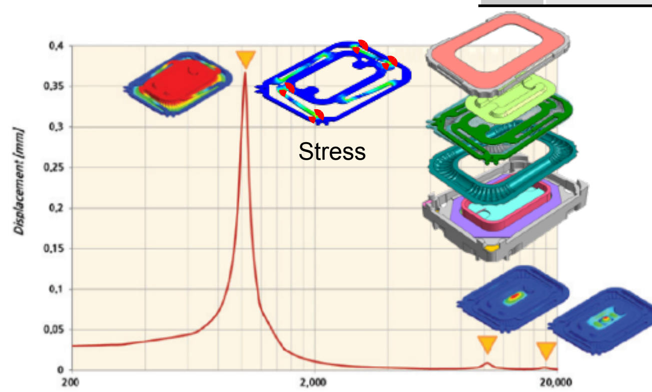
In this project, the performance of a cellphone speaker is evaluated under different sound pressure levels.



Mode	Frequency
1ND	1,046 Hz
2ND	1,778 Hz
3ND	1,801 Hz
4ND	9,457 Hz
5ND	9,679 Hz
6ND	10,217 Hz

Firstly modal analysis was performed to determine natural frequencies of the speaker components. Then frequency response analysis was performed to calculate stresses and deformation shapes of the speaker components according to different frequency spectrums.

From the right image, we can see different mode shapes of the structure. We can also observe that maximum displacement was 0.4 mm, it occurred at around 1000Hz. At this frequency the suspension structure reached its maximum stress of 250MPa.



Displacement / Frequency

Case 2

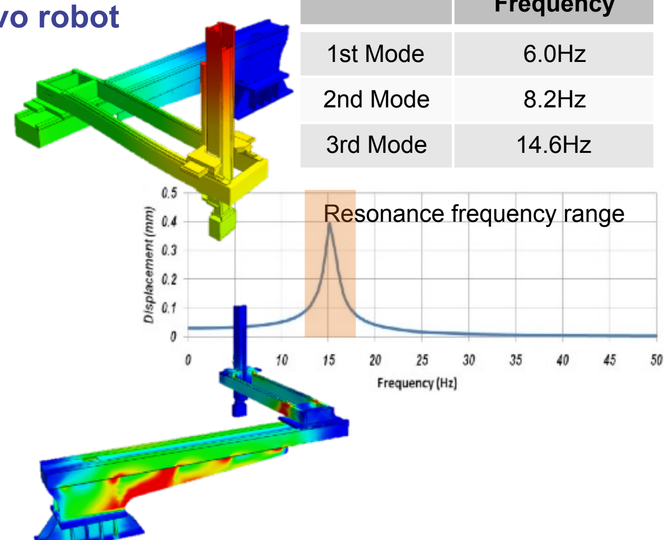
Resonance avoidance of ultra large AC servo robot

Mode	Natural Frequency
1st Mode	6.0Hz
2nd Mode	8.2Hz
3rd Mode	14.6Hz

In this case a dynamic characteristics of a servo robot arm was investigated both to avoid resonance during machine operation and to ensure structural safety during earthquakes.

From the image we can confirm the necessity for resonance avoidance design in order to avoid the resonance which happens at 15Hz under repetitive load.

Stresses were equally calculated under seismic load through response spectrum analysis.



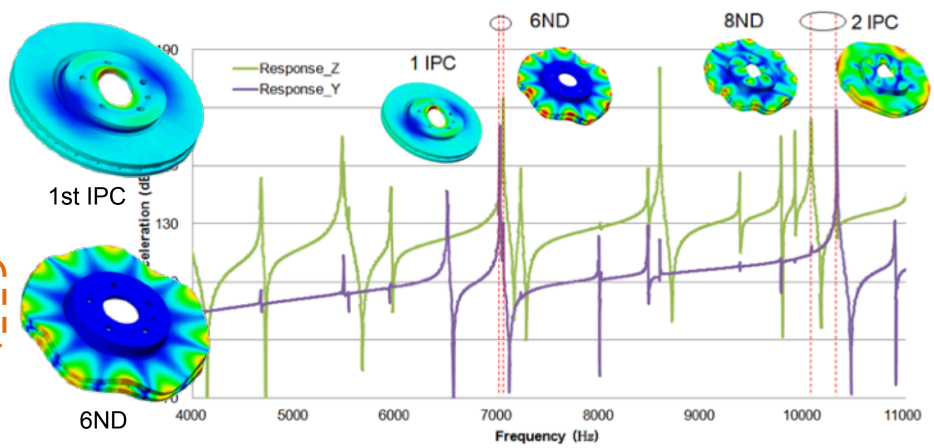
Stress distribution under seismic load

Case 3

Brake Disc Squeal Analysis

In this project the dynamic characteristics of brake disc were reviewed to avoid squeal noise caused by vibration. Through frequency response analysis, we can observe that at around 7000 Hz, frequencies of Nodal Diameter Mode and In-Plane Compression Mode are very close, where squeal noise is most likely to occur. Therefore, a design modification is needed to separate 2 frequencies to avoid squealing problems.

Mode	Natural frequency
2ND	1027 Hz
3ND	2394 Hz
4ND	3897 Hz
5ND	5468 Hz
1st IPC	7014 Hz
6ND	7045 Hz
7ND	8592 Hz
8ND	10068 Hz
2nd IPC	10285 Hz



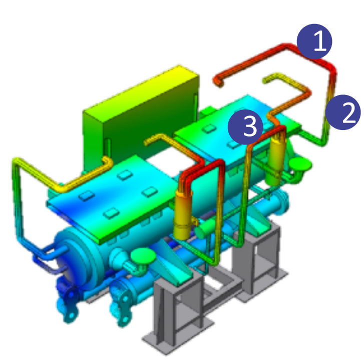
Case 4

Safety analysis of marine refrigeration machine under vibration

In this project, natural frequency analysis and frequency response analysis were performed to predict the happening of cracks on the body and piping of a marine refrigeration machine under vibration loads.

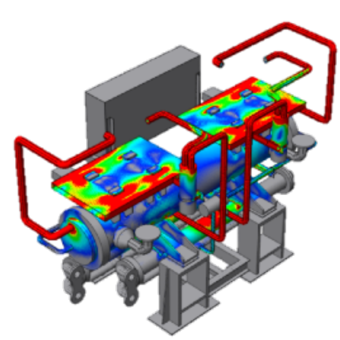
Pipe① Part frequency response

Pipe② Part frequency response



Resonant displacement distribution

Pipe③ Part frequency response



Resonant stress distribution

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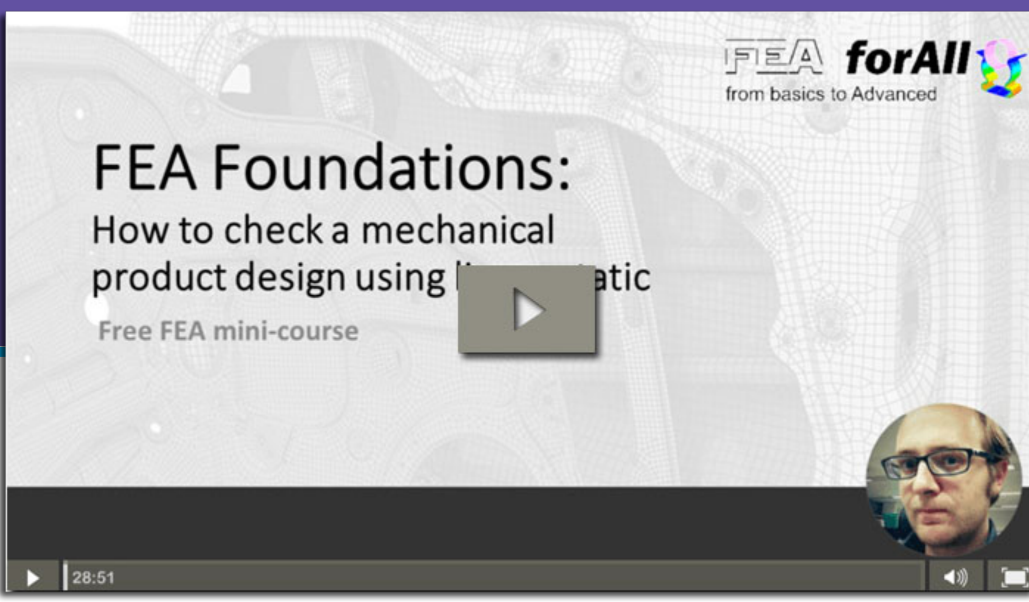

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